Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

4. The probable distribution of colors in the background of N. G. C. 1647, and that actually observed for M 67, is similar to that for the North Pole and the region of S Cygni.

5. In neither case is the limiting magnitude low enough to include all the cluster stars; but as far as these limits, the percentage distribution of the different colors in the cluster is notably different from that in the background; moreover, there is a marked difference for the two clusters. N. G. C. 1647 contains a considerable number of *b* and *a* stars, and practically none of the *g*, *k*, and *m* classes, except as they may be fainter than the limiting magnitude. M 67, on the other hand, shows no *b* and only 3% of *a* stars. The maximum frequency of 71% is for *g* and, curiously enough, no *m* stars appear.

[1] *Mt. Wilson Contr.*, No. 100; *Astrophys. J.*, 42, (1915).

[2] *Mt. Wilson Contr.*, No. 102; *Astrophys. J.*, 42, (1915).

[3] A Notation for Use in the Discussion of Star Colors, these PROCEEDINGS, 1, 481 (1915).

[4] *Mt. Wilson Contr.*, No. 81; *Astrophys. J.*, 39, 361 (1914).

ON THIELE'S 'PHASE' IN BAND SPECTRA

By Horace Scudder Uhler

SLOANE PHYSICAL LABORATORY, YALE UNIVERSITY

Presented to the Academy, July 24, 1915

In the emission spectra of all compounds and of many elements the lines start abruptly at a certain wave-length, near which they are very close together, and separate more and more as the distance from the beginning or 'head' is increased. Such spectra are called band or channeled spectra because, with relatively small dispersion, they have the general appearance of the chiaro-oscuro of a channeled column illuminated laterally. One of the problems of spectroscopy consists in finding the law governing the relative frequencies of the lines of one band or of a group of bands. In other words, the problem is to arrange the lines in series and to express their frequencies by a mathematical formula, as has been done for a fairly large number of line spectra. The analysis of a line spectrum into series is greatly facilitated and made secure by the fact that lines of the same series show their functional relationship by the similarity of the changes which they undergo when the source is subjected to pressure, or is placed in a magnetic field, etc. On the other hand, the frequencies of the lines of the majority of well-developed band spectra do not exhibit the pressure and Zeeman effects. Consequently, since physical criteria are lacking, the grouping into series of the lines of band spectra depends wholly upon the apparent regular sequence of the lines and the degree of accuracy with which their wave-lengths or their frequencies fit a smooth curve or satisfy an empirical equation. It is therefore evident that any formula proposed for band spectra must be subjected to very severe tests before it can be accepted as an expression of a law of nature.

A very general law for all series spectra was proposed by Thiele in the year 1897. It was expressed by the equation

$$\lambda = f[(n + c)^2], \qquad (1)$$

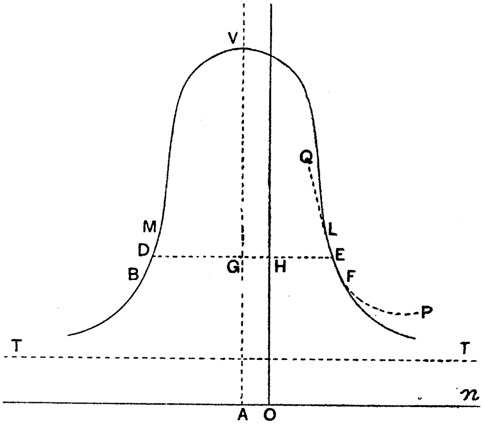
where λ denotes the wave-lengths of a line whose ordinal number is n, and c is a constant for any one series. Thiele styled c the "phase" of the series. The integer n is supposed to assume negative as well as positive values. Consequently a series must in general be composed of two groups of lines each of which would ordinarily be called a series. More precisely, the positive branch of each series must be accompanied by a negative branch of the same series, having the same head ($n = 0$) and the same 'tail' ($n = \infty$) and being represented alternately by a line in each interval of the other branch. In the case of band spectra Thiele highly recommended the special form of function (1) which he employed with apparent success in his computations of the wave-lengths of the lines of the carbon band at λ 5165 and which he wrote as

$$\lambda = \lambda_0 - \kappa \left(\frac{n+c}{10}\right)^2 \cdot \frac{1 + s_1\left(\frac{n+c}{10}\right)^2 + \cdots + s_{r-1}\left(\frac{n+c}{10}\right)^{2r-2}}{1 + t_1\left(\frac{n+c}{10}\right)^2 + \cdots + t_r\left(\frac{n+c}{10}\right)^{2r}} \qquad (2)$$

The objects of the present paper are to outline a practical method for calculating c *without assuming a special form* of function (1) and to give briefly the results obtained by applying the new method to the λ 5165 carbon band and the third cyanogen band at λ 3883.

It should be remarked in advance that it is not feasible to solve equation (2) for c when a sufficiently large number of wave-lengths are used to ensure a satisfactory degree of accuracy. The adjacent figure shows the form of curve representing equation (1) in rectangular coördinates. The even power of $n + c$ involved in (1) causes the graph to be symmetrical with respect to the line AV whose equation is $n = -c$. $AV = \lambda_0$. The branches of the curve are asymptotic to the line TT' whose constant ordinate, $\lambda_0 - \kappa\, s_{r-1}/t_r = \lambda_t$, equals the wave-length of the tail of the series. The points of inflection express the hypothesis that the intervals between consecutive lines of the same series attain a maximum, as n increases arithmetically, and then decrease indefinitely. The curve also brings out two more properties of function (1) which are that the number of lines is finite and infinite respectively in the regions of the head and tail of a band.

A straight line parallel to the axis of n, and at a suitable distance therefrom, will intersect the curve in two points, such as D and E. Hence, if n and n' denote (the algebraic values of) the abscissae of any two points on the curve which have the same value of λ, it follows at once that $c = -\frac{1}{2}(n + n')$. [$GH = c$, $HE = n$, $DH = -n'$.] In the case of any series whose wave-lengths have been accurately determined there is no inherent difficulty associated with the calculation of n and n' corresponding to a chosen numerical wave-length. It is only necessary to evaluate the coefficients of any simple, convenient interpolation formula which represents a curve PQ fitting the locus of actual wave-lengths sufficiently closely over a limited range of spectral lines, such as FL or BM. In other words, the value of c may be obtained by taking an adequate number of terms of the power polynomial $\lambda = a_0 + a_1n + a_2n^2 + \ldots + a_kn^k$, determining the coefficients $a_0, a_1, \ldots, a_k$ from the known wave-lengths, substituting for λ an arbitrary wave-length (OH), and solving for n. Then using the same value of λ and repeating the process for the negative quadrant, the corresponding value of n' is computed. Knowing n and n', c follows immediately from the relation $c = -\frac{1}{2}(n + n')$. In practice I have found that three coefficients (a_0, a_1, a_2) are sufficient for all cases, that the method of least squares can be used to great advantage, and that certain transformations of coördinates simplify the computations enormously.



The accompanying table gives the values of c computed by the above method for two of the series of the λ 5165 carbon band. This band is the one investigated arithmetically by Thiele who obtained mean values of c by applying the laborious method of trial and error to formula (2). The tabulated values of the phase were calculated from the more recent and accurate wave-lengths published by Leinen in the year 1905.

The numbers in the fourth column show conclusively that c is not strictly constant for either the α or the δ series of the λ 5165 carbon band. In the case of the α series the phase increases from about 0.263 to 0.268 at least, while for the δ series it decreases from 0.250 to 0.235 or less. The values of c used by Thiele were 0.266 and 0.2445 for the α and δ series respectively.

The fact that the variations of the phase are real and not fictitious may be established by the following considerations. Each of the seven lines +36 to +42 would have to be displaced in the same direction, relatively to the corresponding group of lines (−37 to −43), by 0.104 A to change the value of c from 0.2686 to 0.2629. This shift is too great to be admitted, for, the *arithmetical sum* of all the fourteen residuals ($\lambda_{calc.} - \lambda_{obs.}$) divided by 14 only equals 0.007 A. In like manner, for the δ series, the average residual is 0.016 A whereas the displacement required to change c from 0.2344 to 0.2499 would be 0.355 A. By applying the same method and tests to the two apparent series of doublets which start at the first head of the λ3883 cyanogen band I have been able to show that these series cannot be combined as the positive and negative branches of a single Theile series with a constant phase.

SERIES	+ BRANCH . n .	− BRANCH . n'.	c	λ INTERVAL
α.........	+ 3,...,+ 9	− 3,...,− 9	0.2632	5164.4−5153.4
α.........	+ 3,...,+ 9	− 4,...,−10	0.2633	
α.........	+ 9,...,+16	− 9,...,−16	0.2626	5155.7−5129.8
α.........	+ 9,...,+16	−10,...,−17	0.2625	
α.........	+20,...,+26	−20,...,−27	0.2666	5116.0−5075.5
α.........	+36,...,+42	−36,...,−42	0.2684	5008.0−4944.9
α.........	+36,...,+42	−37,...,−43	0.2686	
δ.........	+ 8,...,+18	− 8,...,−18	0.2499	5156.3−5119.4
δ.........	+23,...,+29	−23,...,−29	0.2417	5095.4−5052.8
δ.........	+35,...,+41	−35,...,−41	0.2375	5009.6−4951.6
δ.........	+52,...,+60	−52,...,−60	0.2344	4843.2−4746.6

In conclusion, it may not be superfluous to remark that equation (2) is unsatisfactory in two respects: (a) as an interpolation formula it is too complicated, since modern wave-lengths require the retention of at least eight parameters, and (b) it predicts tails much too far away from the heads and in a region of the ultra-violet where no tails have been found experimentally. For additional details and comments reference must be made to the *Astrophysical Journal* in which the present paper will be presented in full.